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THE ACCOUNT FOR THE EFFECT OF PHYSICAL PROPERTIES IN HEAT- AND MASS-TRANSFER PHENOMENA

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Аннотация—В работе изложен общий метод учета влияния физических свойств при обобщении коэффициентов теплообмена. На основе совместного рассмотрения системы интегро-дифференциальных уравнений, описывающих явление переноса и системы аналитических связей, описывающих закон соответственных состояний для физических свойств, дается общее уравнение, связывающее основные режимные параметры и главные термодинамические характеристики среды.

NOMENCLATURE

$K_1 \dots K_n$,	similarity numbers obtained from the analysis of the system of equations describing transfer phenomena;
$X_1 \dots X_n$,	physical properties of heat carriers;
$\beta_1 \dots \beta_i$,	regime parameters included into the conditions of uniqueness—rate, geometry, heat flux, etc.;
$\alpha_{P,T}$,	transfer coefficient at parameters P, T;
P,	current pressure;
P_{cr},	critical pressure of the medium;
T,	current temperature;
T_{cr},	critical temperature of the medium;
M,	molecular weight of the medium;
R,	universal gas constant;
g,	gravitational acceleration;
C_v,	heat capacity of the ideal gas state ($P \to 0$);
q,	heat flux on the heating surface;
l,	coordinate;
Π,	parachor;
e,	base of natural logarithms;
n,	exponent and serial number.

As a rule the coefficients of convective transfer are generalized on the basis of the analysis of the system of integro-differential equations and other relations describing transfer phenomena, including physical properties of heat carriers together with regime parameters.

The dependence of the physical properties upon the temperature, pressure and other parameters is usually considered without any general interrelation with the basic equations of the process.

In the light of a number of recently published works [1, 5–10] we can formulate a general method to allow for the physical properties when correlating the coefficients of convective transfer. This method is based on a simultaneous solution of the system of equations describing both the transfer phenomena and conditions of modelling and interrelation between physical properties of the heat carriers.

The integro-differential system of equations describing transfer processes in the light of modern theoretical, experimental and computing methods leads to the solution of the form [2, 3, 4]

$$F(K_1 \dots K_n) = 0. \qquad (1)$$

In this equation the dimensionless groups $K_1 \dots K_n$ determine some set of various regime parameters of transfer phenomena and physical properties of heat carriers.

The system of equations describing in quite a general form the interrelation between physical properties of mass- and heat-carriers and main physico-thermodynamic parameters is determined by the law of corresponding states, and in the light of modern molecular theory of liquids and gases [10] can be written in the form

$$F\left(X_i, P_{cr}, T_{cr}, M, g, R, \frac{P}{P_{cr}}, \frac{T}{T_{cr}}, \frac{C_v}{R}\right) = 0. \tag{2}$$

This general relation for the particular physical property X_i at parameters P, T will be written in the form [1, 6]

$$X_{P,T} = \phi(M, R, g, P_{cr}, T_{cr})\,\psi\left(\frac{P}{P_{cr}}, \frac{T}{T_{cr}}, \frac{C_v}{R}\right). \tag{3}$$

For the saturation line $P = f(T)$ and neglecting the effect of the adiabatic exponent of the ideal gas state (C_v/R) this equation becomes simpler and is changed to the form

$$X_P = \phi_1(M, R, g, P_{cr}, T_{cr})\,\psi_1\left(\frac{P}{P_{cr}}\right) \tag{4}$$

or

$$X_T = \phi_2(M, R, g, P_{cr}, T_{cr})\,\psi_2\left(\frac{T}{T_{cr}}\right). \tag{4a}$$

Here it is necessary to note that the function ϕ from equations (3–4a) has the dimensions of the left hand side of the equation, since the function ψ is dimensionless.

Matching of systems (1) and (2) leads to a more general solution of the problem corresponding to the closed system of initial equations including not only the regime parameters but also the influence of the physical properties upon the convective heat- and mass-transfer phenomena.

It is useful to detail the above general scheme in the following way.

System (1) can be written in the form

$$K_1 = f(K_2 \dots K_n). \tag{5}$$

Grouping the physical properties of the medium in the right-hand part, we have

$$\frac{\alpha_{P(T)}}{\beta_1^{n_1} \dots \beta_i^{n_i}} = f_1(X_1 \dots X_n). \tag{6}$$

In accordance with equation (3) in the general case equation (6) is transformed to the form

$$\frac{\alpha_{P,T}}{\beta_1^{n_1} \dots \beta_i^{n_i}} = \phi_\Sigma(M, R, g, P_{cr}, T_{cr}) \times \psi_\Sigma\left(\frac{P}{P_{cr}}, \frac{T}{T_{cr}}, \frac{C_v}{R}\right). \tag{7}$$

For the saturation line and using equations (4) and (4a) we have

$$\frac{\alpha_P}{\beta_1^{n_1} \dots \beta_i^{n_i}} = \phi_{\Sigma_1}(M, R, g, P_{cr}, T_{cr})\,\psi_{\Sigma_1}\left(\frac{P}{P_{cr}}\right) \tag{8}$$

$$\frac{\alpha_T}{\beta_1^{n_1} \dots \beta_i^{n_i}} = \phi_{\Sigma_2}(M, R, g, P_{cr}, T_{cr})\,\psi_{\Sigma_2}\left(\frac{T}{T_{cr}}\right). \tag{8a}$$

In these equations the subscript Σ indicates a complicated nature of the functions ϕ and ψ which reflects a certain spectrum of values of physical properties X_i expressed by the group of the same physico-thermodynamic parameters of the system.

The obtained equation (7) is a completely general simultaneous solution of the initial systems (1) and (2) and can be considered as that providing the physically closed character of the single system describing the transfer phenomenon with variable physical properties of the media.

As an example apply system (8) to generalize the data on heat transfer in boiling and condensation.

In this case the left hand side of equation (8), when use is made of the known experimental facts, will be of the form

$$\frac{\alpha_P}{\beta_1^{n_1} \dots \beta_i^{n_i}} \equiv \frac{\alpha_P}{q^{\frac{2}{3}}}, \tag{9}$$

since the developed nucleate boiling is characterized only by one regime parameter, viz. the heat flux q at the heating surface.

Accordingly equation (8) is transformed to the form

$$\alpha_P = q^{\frac{2}{3}} \phi_{\Sigma_1}(P_{cr}, T_{cr}, M, g, R)\, \psi_{\Sigma_1}\left(\frac{P}{P_{cr}}\right). \qquad (10)$$

Comparing this with the experimental data [7, 8, 11] by using specially developed methods one obtains the following generalized relation for heat-transfer coefficients in developed nucleate boiling

$$\alpha_P = 384\, q^{\frac{2}{3}}\, P_{cr}{}^{\frac{1}{3}}\, T_{cr}{}^{-\frac{5}{6}}\, M^{-\frac{1}{6}} \times \left(\frac{P}{P_{cr}}\right)^{\frac{1}{10}} \left[1 + 4{\cdot}65\left(\frac{P}{P_{cr}}\right)^{1{\cdot}16}\right] \frac{\text{kcal}}{\text{m}^2\,\text{h}\,\text{degC}}. \qquad (11)$$

A dependence somewhat different in its form can also be derived when the parameter T/T_{cr} rather than P/P_{cr} is selected.

The universal constants R and g are included in the numerical coefficient in formula (11).

The same was done to correlate heat transfer in vapour condensation [9] and burn-outs in developed nucleate boiling under the free convection conditions [1, 12, 13, 14].

For condensation of motionless vapour on the horizontal surface general relation (8) is transformed to the form

$$\bar{\alpha}_P = 1{\cdot}55 \times 10^5\, P_{cr}{}^{\frac{4}{9}}\, T_{cr}{}^{-\frac{2}{9}}\, M^{-\frac{2}{3}}\, q^{-\frac{1}{3}}\, l^{-\frac{1}{3}}\, F \times \left(\frac{P}{P_{cr}}\right) \text{kcal/m}^2\,\text{h}\,\text{degC}. \qquad (12)$$

Different functions of the relative pressure $F(P/P_{cr})$ are given here for the three ranges of values P/P_{cr} due to requirements of the algebraic selection of a single curve

(a) at $P/P_{cr} \leqslant 0{\cdot}01$

$$F\left(\frac{P}{P_{cr}}\right) = 0{\cdot}5 + \frac{P/P_{cr}}{0{\cdot}00107 + 1{\cdot}965\, P/P_{cr}};$$

(b) at $0{\cdot}01 \leqslant P/P_{cr} \leqslant 0{\cdot}1$

$$F\left(\frac{P}{P_{cr}}\right) = 1;$$

(c) at $0{\cdot}1 \leqslant P/P_{cr} < 1$

$$F(P/P_{cr}) = 1{\cdot}05 - \frac{P/P_{cr}}{1{\cdot}5 + 0{\cdot}5\, P/P_{cr}}.$$

The analysis of the experimental data on burn-outs in nucleate boiling in a large volume based on the above theory led to the generalized calculation formula of the form [1, 12, 13, 14]

$$q_{cr} = \left[2{\cdot}8\left(\frac{P}{P_{cr}}\right)^{0{\cdot}3} - 0{\cdot}17 \exp\left(3\frac{P}{P_{cr}}\right) + 0{\cdot}23\right] \times P_{cr}\left[\sqrt{\frac{RT_{cr}}{M/g}}\right] \times (0{\cdot}06 + 0{\cdot}75 \times 10^{-3}\,\Pi) \quad \text{kcal/m}^2\text{h}. \qquad (13)$$

The value of parachor Π is calculated by the structural chemical formula of the substance [10]. For some of the most widely used substances the values of molecular weights, parachors and critical parameters are given in the table.

As the comparison with experiments and various calculations shows [1, 7, 9, 11, 14, 15], the relations obtained cover a much greater set

Table 1

Liquid	Chemical formula	P_{cr} (kg/cm^2)	T_{cr} (°K)	M (kg/mol)	Parachor Π
Water	H_2O	225·65	647	18·02	54·0
Methyl alcohol	CH_3OH	81·10	513	32·04	88·5
Ethyl alcohol	C_2H_5OH	65·10	516	46·07	132·0
Propyl alcohol	C_3H_7OH	51·8	537	60·10	166·0
Benzol	C_6H_6	49·50	562	78·11	206·0
Heptane	C_7H_{16}	27·9	540	100·21	309·0

of heat carriers in comparison with the available criterial formulae and give much better correlation of experimental data.

To account for the physical properties when correlating the transfer coefficient, the similar correlation can be done not only along the saturation line but also in the most general case with simultaneous account for both coordinates (P, T) in accordance with equation (7). Some steps in this direction have already been done in [16].

CONCLUSIONS

A general method is proposed for the account of the physical properties in heat- and mass-transfer phenomena. This method consists in including the law of corresponding states written in the most general form (2) into equations of the process (1). This leads to the relation between systems (1) and (2) expressed in the form of equations (8), (11), (12) and (13). The latter relations are quite general solutions of initial systems (1) and (2) for various phenomena of heat and mass transfer and can be considered as those providing closeness of a single system describing transfer phenomena with variable properties of the media.

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Abstract—The paper presents a general method of accounting for the effect of the physical properties when correlating heat-transfer coefficients. On the basis of a simultaneous consideration of the system of integro-differential equations describing the transfer phenomenon, and the system of analytical relations describing the law of corresponding states for the physical properties, a general equation is given relating the main regime parameters and the principle thermodynamic properties of the medium.

Résumé—L'article présente une méthode générale pour tenir compte de l'effet des propriétés physiques dans la corrélation des coefficients de transport de chaleur. On donne, en se basant sur un examen simultané du système des équations intégro-différentielles décrivant le phénomène de transport, et du système des relations théoriques décrivant la loi des états correspondants pour les propriétés physiques, une équation générale reliant les paramètres principaux du régime et les propriétés thermodynamiques principales du milieu.

Zusammenfassung—Die Arbeit liefert eine algemeine Methode zur Berücksichtigung des Einflusses der Stoffwerte bei der Korrelation von Wärmeübergangskoeffizienten. Aufgrund gleichzeitiger Betrachtungen des Systems von Integro-Differentialgleichungen zur Beschreibung des Übergangsphänomens und des Systems analytischer Beziehungen zur Beschreibung des Gesetzes korrespondierender Zustände für die Stoffwerte wird eine allgemeine Gleichung angegeben für den Zusammenhang von Hauptparametern des Regimes und prinzipiellen thermodynamischen Stoffwerten.